Short Lecture

Power Theory: Power Currents, Active Currents, Nonactive Currents

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To be used only with the lecture!

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1 Introduction

In case of d.c. circuits the product of voltage and current of a load is always equal to the power taken or given by this load. Already in the beginnings of a.c. circuits it was recognised, that the product of the rms values of voltage and current may be larger than the power of the load. Therefore this product was called apparent power.

Already in 1865 the concept of phase displacement was first introduced by Maxwell [1]. In 1892 Steinmetz [2] pointed out, that in case of nonlinear loads nonactive currents can be observed, which do not contribute to energy transfer. A lot of contributions from many authors followed, which are listed in a historical overview published in 1979 by Skudelny [3].

In contrast to power - a quantity which is physically defined - apparent power by default is not a physically based quantity, but a conventional one. Therefore power definitions have always been - and are still - a matter of discussion.

For quite a long time discussion concerning to only few European countries. Due to the extensive use of nonlinear elements in converters, power supplies and many other devices the discussion has been rising again on an international basis [4, 5], leading to a publication agreed upon by many experts in the field [6].

Other well known power theories are these published by Budeanu [7] and Quade [8]. The Budeanu theory is based on the decomposition of current and voltage into harmonics. In case of nonsinusoidal waveforms much effort is needed for the calculations. The results received from Budeanu theory are not always sensible [9, 10]. Quade's theory is based on conservative quantities and - as a theory - correct and consistent. The multi-terminal circuit is seen as a multi-port structure. If the power of each of the ports - seen for itself - is purely active, then the nonactive power of the complete multi-terminal circuit vanishes, no matter if the active power is evenly distribute or not. Moreover, Quade's theory is not invariant to changes of the current-voltage-system [11, 12].

In this short lecture I want to describe the power definitions going back to Fryze [13] and Buchholz [14, 15, 16], which have been extended by Depenbrock to the so-called FBD-method [17, 18, 19]. The FBD-method - and therefore this lecture - includes the definitions agreed upon in [6]. It is the only known set of power definitions which can unambiguously treat every conceivable circuit with arbitrary number of terminals irrespective of its structure or behaviour, linear and nonlinear. The definitions given in the revised and upcoming German standards concerning power definitions [20, 21] are the same as those contained in the FBD-method.

2 General definitions

2.1 Currents and voltages of an *n*-terminal circuits

In this lecture we will discuss the very general concept of an *n*-terminal circuit. Such an *n*-terminal circuit is depicted in Fig. 2.1:

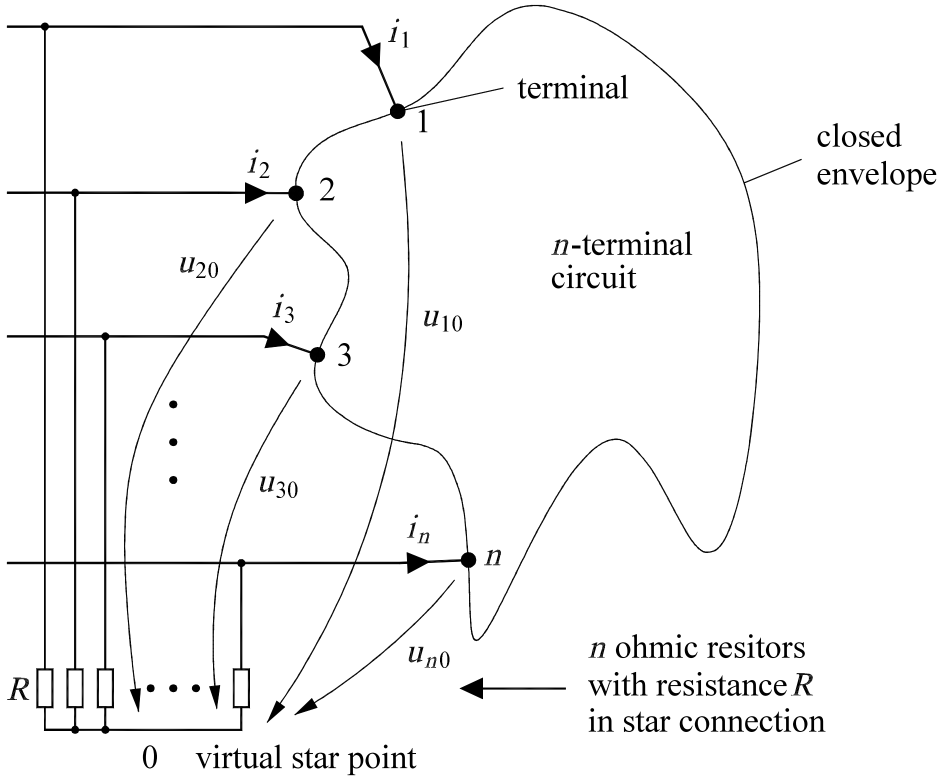


Figure 2.1: n-terminal circuit: terminals and associated currents and voltages

Some restrictions must be ascertained by the closed envelope and the terminals:

- no radiation is allowed, energy transfer is restricted to the terminals
- no distributed quantities (as e.g. current densities) are allowed. If for example the earth current in connection with overhead power supply lines has to be discussed, we have to
  - go to a terminal (transformer, substation)
  - collect the current to a lumped quantity (by integration) and associate a potential to this artificial terminal which keeps the energy transfer (the power) associated with the distributed quantity constant

Care must also be taken when selecting the terminals to be taken into consideration. Circuits often have terminals intended for the main energy transfer and other terminals intended only for protection purposes or control purposes. Only those terminals intended for the main energy transfer are to be assessed, not protective terminals or control terminals. This includes the terminal for the neutral conductor but excludes the terminal for the protective earth conductor.

According to Kirchhoff's laws the *n* currents of such an *n*-terminal circuit always sum up to zero:

$$\sum_{\upsilon=1}^{n} i_\upsilon(t) = 0\,; \quad \upsilon \in \{1, 2, \cdots, n\}\ . \tag{2.1}$$

Such quantities will from here on be called zero-sum quantities.

Clearly, only n-1 currents are linearly independent; if they are known, the remaining current can easily be determined based on (2.1).

The n voltages of the circuit may be measured against an arbitrary reference potential. In practice, the potential of one of the terminals is often selected as reference potential, because in this case only n-1 voltages have to be measured, the nth voltage is then always zero. It follows that only n-1 voltages are independent. It is advisable - as will be stressed in the next section when discussing current decomposition and powers - to choose a specially selected fixed reference potential for the theory. This reference potential is associated with the so-called virtual star point "0" realised by a star connection of n equal ohmic resistors with resistance R. The n voltages $u_{\upsilon 0}$ between the n terminals and the virtual star point are then zero-sum quantities and thus have the same property as the currents:

$$\sum_{\upsilon=1}^{n} u_{\upsilon 0}(t) = 0\,; \quad \upsilon \in \{1, 2, \cdots, n\}\ . \tag{2.2}$$

It is easy to show that (2.2) holds true: The currents flowing through the n star-connected resistors are zero-sum quantities by Kirchhoff's laws. The voltages are derived by multiplying all currents with the same resistance R which can be placed in front of the summation symbol.

In practice it is not necessary to realise the virtual star point. The zero-sum voltages $u_{\upsilon 0}$ can also be calculated from other measured voltages, for example based on terminal to terminal voltages:

$$u_{\upsilon 0}(t) = \frac{1}{n} \sum_{\mu=1}^{n} u_{\upsilon\mu}(t)\,; \quad \upsilon \in \{1, 2, \cdots, n\}\ . \tag{2.3}$$

Another practically relevant possibility is to measure the $n-1$ voltages against one of the conductors, e.g. $u_{\upsilon n}$ against the n-th conductor, and calculate the voltage against the virtual star point by adding the voltage u_{n0} between this n-th conductor and the virtual star point:

$$\begin{aligned} u_{\upsilon 0}(t) &= u_{\upsilon n}(t) + u_{n0}(t) \\ 0 &= \sum_{\upsilon=1}^{n} u_{\upsilon 0}(t) = \sum_{\upsilon=1}^{n} [u_{\upsilon n}(t) + u_{n0}(t)] = \sum_{\upsilon=1}^{n} u_{\upsilon n}(t) + \sum_{\upsilon=1}^{n} u_{n0}(t) = \sum_{\upsilon=1}^{n} u_{\upsilon n}(t) + n \cdot u_{n0}(t)\ . \\ &\Leftrightarrow u_{n0}(t) = -\frac{1}{n} \sum_{\upsilon=1}^{n} u_{\upsilon n}(t) \end{aligned} \tag{2.4}$$

Based on the zero-sum voltages $u_{\upsilon 0}$ measured against the virtual star point voltages measured against an arbitrary reference potential r can be introduced: Let $u_{0\mathrm{r}}$ be the voltage between the virtual star point, then the n voltages against the reference potential are given by

$$u_{\upsilon \mathrm{r}}(t) = u_{\upsilon 0}(t) + u_{0\mathrm{r}}(t)\,; \quad \upsilon \in \{1, 2, \cdots, n\}\ . \tag{2.5}$$

2.2 Collective instantaneous values

Collective instantaneous values are introduced to characterise a set of zero-sum quantities. They will prove to be extremely useful when discussing power and current components. The collective instantaneous value associated with a set of zero-sum quantities is identical with the (Euclidean) norm of a vector whose components are given by the zero-sum quantities. For arbitrary quantities g_υ the collective instantaneous value g_Σ is given by

$$g_\Sigma(t) = \left\| \begin{bmatrix} g_1(t) & g_2(t) & \cdots & g_n(t) \end{bmatrix}^{\mathrm{T}} \right\| = \sqrt{\sum_{\upsilon=1}^{n} g_\upsilon^2(t)}\ . \tag{2.6}$$

It reduces the information contained in the n zero-sum quantities to one characteristic quantity, but it is still time dependent. During one period still an infinite number of values may occur. In the same manner as the rms value is defined, a collective rms value $g_{\Sigma\mathrm{rms}}$ can be defined leading to one single characteristic quantity representing the whole set of n periodic zero-sum quantities seen over a whole period T. The collective rms value is given by

$$g_{\Sigma\mathrm{rms}} = \sqrt{\frac{1}{T}\int_{t-T}^{t} g_{\Sigma}^{2}(\tau)\,d\tau} = \sqrt{\frac{1}{T}\int_{t-T}^{t}\sum_{\upsilon=1}^{n} g_{\upsilon}^{2}(\tau)\,d\tau} = \sqrt{\frac{1}{T}\sum_{\upsilon=1}^{n} g_{\upsilon\mathrm{rms}}^{2}}\ , \qquad (2.7)$$

where $g_{\upsilon\mathrm{rms}} = \sqrt{\frac{1}{T}\int_{t-T}^{t} g_{\upsilon}^{2}(\tau)\,d\tau}$ is the well-know rms value of quantity $g_{\upsilon}(t)$.

Both quantities, g_{Σ} and $g_{\Sigma\mathrm{rms}}$, can only be zero or positive. If the collective instantaneous value is zero, then all n zero-sum quantities are zero in that time instant. If the collective rms value is zero, all quantities are permanently zero.

It should be noted that only zero-sum quantities may be used to calculate collective quantities. Any deviation from the zero-sum property will increase the collective quantity. Therefore only voltages related to the unambiguously defined virtual star point (2.3, 2.4) may be used to calculate collective quantities of voltages.

Collective values in the way defined here are justified by a theoretical and a practical consideration. The theoretical justification of this concept is given by their direct relation to the Cauchy-Schwarz inequalitites as seen in section 3.2 for instantaneous quantities and section 4.2 for periodic quantities. The practical justification is given by the aspect of minimising line losses under the assumption of equal resistances in each feeding conductor: Consider that the n-terminal circuit seen in Fig. 2.1 is fed by n conductors with equal resistances R_{line}. Then the instantaneous power consumed by these resistances (the total instantaneous line loss) is given by

$$p_{\Sigma\mathrm{loss}}(t) = \sum_{\upsilon=1}^{n} R_{\mathrm{line}} \cdot i_{\upsilon}^{2}(t) = R_{\mathrm{line}} \sum_{\upsilon=1}^{n} i_{\upsilon}^{2}(t) = R_{\mathrm{line}} \cdot i_{\Sigma}^{2}(t)\ . \qquad (2.8)$$

Minimising the squared instantaneous collective value of currents $i_{\Sigma}^{2}(t)$ will minimise the instantaneous line losses. If all quantities are periodical with period T, then the total line losses can be evaluated as mean value of the instantaneous line losses:

$$P_{\Sigma\mathrm{loss}} = \frac{1}{T}\int_{t-T}^{t}\sum_{\upsilon=1}^{n} R_{\mathrm{line}} \cdot i_{\upsilon}^{2}(\tau)\,d\tau = R_{\mathrm{line}}\frac{1}{T}\int_{t-T}^{t}\sum_{\upsilon=1}^{n} i_{\upsilon}^{2}(\tau)\,d\tau = R_{\mathrm{line}} \cdot i_{\Sigma,\mathrm{rms}}^{2}\ . \qquad (2.9)$$

Minimising the squared collective rms value of currents $i_{\Sigma\mathrm{rms}}^{2}$ will minimise total line losses.

A way to adapt this quantity to nonperiodic and nonstationary conditions is discussed later.

3 Instantaneous power and current definitions

3.1 Instantaneous power

The power associated with one terminal of the multi-terminal circuit is in general given by the voltage $u_{\upsilon r}$ of the terminal, measured against an arbitrary reference point (which has to be the same for all terminals), multiplied with the current i_υ of the terminal:

$$p_\upsilon = u_{\upsilon r}\, i_\upsilon = (u_{\upsilon 0} + u_{0r})\, i_\upsilon = u_{\upsilon 0}\, i_\upsilon + u_{0r}\, i_\upsilon \;. \tag{3.1}$$

Clearly, each individual power depends on the voltage u_{0r} between the virtual star point and the selected reference point. Under these circumstance one single terminal power carries no usable information. The sum p_Σ of all terminal powers is of course independent of the selected reference point and leads to the total instantaneous power of the circuit:

$$\begin{aligned} p_\Sigma(t) &= \sum_{\upsilon=1}^{n} p_\upsilon(t) = \sum_{\upsilon=1}^{n} i_\upsilon(t)\cdot u_{\upsilon r}(t) \\ &= \sum_{\upsilon=1}^{n} i_\upsilon(t)\cdot u_{\upsilon 0}(t) + u_{0r}(t)\cdot \sum_{\upsilon=1}^{n} i_\upsilon(t) \\ &= \sum_{\upsilon=1}^{n} i_\upsilon(t)\cdot u_{\upsilon 0}(t) \qquad \text{because} \qquad \sum_{\upsilon=1}^{n} i_\upsilon(t) = 0 \end{aligned} \quad . \tag{3.2}$$

It is evident that the voltage u_{0r} between the virtual star point and the selected reference point is cancelled when calculating total instantaneous power, because the sum of all currents is permanently zero.

These findings clearly support to select the virtual star point as reference point for voltage measurement. Only then currents and voltages are both zero-sum quantities and the power of a single terminal is unambiguously defined and of use.

3.2 Cauchy-Schwarz inequality defines instantaneous power conductance

Let us assume two sets $g_{a,\upsilon}$ and $g_{b,\upsilon}$ of n zero-sum quantities each. The Cauchy-Schwarz inequality is then given by

$$\sqrt{\left(\sum_{\upsilon=1}^{n} g_{a\upsilon}^2(t)\right)\cdot\left(\sum_{\upsilon=1}^{n} g_{b\upsilon}^2(t)\right)} = g_{a\Sigma}(t)\cdot g_{b\Sigma}(t) \geq \left|\sum_{\upsilon=1}^{n} [g_{a\upsilon}(t)\cdot g_{b\upsilon}(t)]\right|. \tag{3.3}$$

The two expressions in (3.3) are only then equal, if

$$g_{a\upsilon}(t) = k(t)\cdot g_{b\upsilon}(t) \quad \forall\ \upsilon \in \{1\cdots n\} \tag{3.4}$$

is fulfilled.

If we associate the set $g_{a,\upsilon}$ with the currents and the set $g_{b,\upsilon}$ with the voltages of an n-terminal circuit, we receive

$$s_\Sigma(t) = u_\Sigma(t)\cdot i_\Sigma(t) \geq \left|\sum_{\upsilon=1}^{n} i_\upsilon(t)\cdot u_{\upsilon 0}(t)\right| = |p_\Sigma(t)|, \tag{3.5}$$

where the left-hand part of (3.3) leads to the definition of the instantaneous apparent power $s_\Sigma(t)$, which is always larger or equal to the total instantaneous power $|p_\Sigma(t)|$ following directly from the right-hand part of (3.3). Instantaneous apparent power and total instantaneous power are only then equal, if **all** n voltages and the associated currents are **linked with the same** factor $k(t) = G_p(t)$ called equivalent power conductance. If this condition is not fulfilled, then the instantaneous apparent power is larger than the total instantaneous power.

This definition of instantaneous apparent power links this conventional quantity as strongly as possible with physical definitions. Assuming given collective instantaneous values for the voltages and currents the instantaneous apparent power is the maximal instantaneous power possible. The definition is constructive, from (3.4) the exact condition under which this maximal instantaneous power is reached follows:

$$i_\upsilon(t) = G_\mathrm{p}(t) \cdot u_{\upsilon 0}(t) \quad \forall\, \upsilon \in \{1 \cdots n\}\ . \tag{3.6}$$

All voltages and associated currents are linked by the equivalent power conductance $G_\mathrm{p}(t)$. If this definition is used for controlling a compensator, an equivalent load results which is discussed in subsection 5.3.

What does this mean in practice? Let us assume that the voltages are given - which is an acceptable assumption in most practical applications. Only if the currents fulfil (3.6), energy transfer is optimal. Otherwise the existing currents can be changed such, that (3.6) is fulfilled and the total instantaneous power remains unchanged. Again the theory is constructive. The total instantaneous power under the condition of (3.6) is

$$p_\Sigma(t) = \sum_{\upsilon=1}^{n} \left[u_{\upsilon 0}(t) \cdot i_\upsilon(t)\right] = \sum_{\upsilon=1}^{n} \left[u_{\upsilon 0}(t) \cdot G_\mathrm{p}(t) \cdot u_{\upsilon 0}(t)\right] = G_\mathrm{p}(t) \sum_{\upsilon=1}^{n} u_{\upsilon 0}^2(t) = G_\mathrm{p}(t) \cdot u_\Sigma^2\ . \tag{3.7}$$

We directly receive the optimal value of the instantaneous power conductance $G_\mathrm{p}(t)$

$$G_\mathrm{p}(t) = \frac{p_\Sigma(t)}{u_\Sigma^2(t)}\ . \tag{3.8}$$

In a similar way $G_\mathrm{p}(t)$ can be defined if the currents are given:

$$G_\mathrm{p}(t) = \frac{i_\Sigma^2(t)}{p_\Sigma(t)}\ . \tag{3.9}$$

3.3 Power currents and powerless currents

Let us again assume that the voltages are given. Then the optimal set of instantaneous currents - called (instantaneous) power currents i_p - follows from (3.6) using the instantaneous power conductance $G_\mathrm{p}(t)$ derived in (3.8)

$$i_{\mathrm{p}\upsilon}(t) = G_\mathrm{p}(t) \cdot u_{\upsilon 0}(t)\ . \tag{3.10}$$

If a difference exists between the set of power currents $i_{\mathrm{p}\upsilon}(t)$ and the set of terminal currents $i_\upsilon(t)$, then energy transfer is not optimal. Power currents - with reduced collective instantaneous value - are sufficient to transfer the same energy. Losses in the feeding conductors can be reduced (assuming equal resistances in all feeding conductors).

The difference between the set of power currents $i_{\mathrm{p}\upsilon}(t)$ and the set of terminal currents $i_\upsilon(t)$, if it exists, is called set of powerless currents $i_{\mathrm{ix}\upsilon}(t)$

$$i_{\mathrm{ix}\upsilon}(t) = i_\upsilon(t) - i_{\mathrm{p}\upsilon}(t)\ , \tag{3.11}$$

in other words: The terminal currents are the sum of power currents and powerless currents:

$$i_\upsilon(t) = i_{\mathrm{p}\upsilon}(t) + i_{\mathrm{ix}\upsilon}(t)\ . \tag{3.12}$$

It should be noted that in literature often the term "instantaneous active currents" is used for power currents. This should strictly be avoided because of reasons detailed in subsection 4.4.

If we now calculate the total instantaneous power associated with the powerless currents, we receive

$$p_\Sigma(t) = \sum_{\upsilon=1}^{n} \left[u_{\upsilon 0}(t) \cdot i_{\mathrm{ix}\upsilon}(t)\right] = \sum_{\upsilon=1}^{n} \left[u_{\upsilon 0}(t) \cdot i_\upsilon(t)\right] - \sum_{\upsilon=1}^{n} \left[u_{\upsilon 0}(t) \cdot i_{\mathrm{p}\upsilon}(t)\right] = 0\ , \tag{3.13}$$

because the power currents were defined such, that they transfer the same energy as the terminal currents.

3.4 Vectorial notation

Up to now we have always used the currents and voltages separately, utilising the summation symbol in some of the equations to incorporate all n quantities into a single characterising quantity. Looking closely at the equations, it is obvious that vectors can be used efficiently to decrease the writing effort and increase graphic understanding.

We now associate - as already hinted at in connection with (2.6) - the n zero-sum voltages with the components of a voltage vector $\boldsymbol{u}$, and the n zero-sum currents with the associated components of a current vector $\boldsymbol{i}$. The collective instantaneous values are then given by the (Euclidean) norm of these vectors, which is the square root of the scalar product of each of the vectors with itself:

$$u_\Sigma(t) = \|\boldsymbol{u}(t)\| = \sqrt{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t)} \ ; \ i_\Sigma(t) = \|\boldsymbol{i}(t)\| = \sqrt{\boldsymbol{i}(t) \cdot \boldsymbol{i}(t)} \ . \tag{3.14}$$

The apparent power is the product of the collective instantaneous values of voltages and currents (3.5):

$$s_\Sigma = u_\Sigma(t) \cdot i_\Sigma(t) = \|\boldsymbol{u}(t)\| \cdot \|\boldsymbol{i}(t)\| = \sqrt{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t)} \cdot \sqrt{\boldsymbol{i}(t) \cdot \boldsymbol{i}(t)} \ . \tag{3.15}$$

The instantaneous power results from the scalar product of current vector and voltage vector:

$$p_\Sigma(t) = \boldsymbol{u}(t) \cdot \boldsymbol{i}(t) \ . \tag{3.16}$$

Optimal energy transfer is characterised by

$$\boldsymbol{i}(t) = G_\mathrm{p}(t) \cdot \boldsymbol{u}(t) \ , \tag{3.17}$$

in this case the vector of currents and the vector of voltages have the same (or, in case of negative values of the instantaneous power conductance $G_\mathrm{p}(t)$ opposite) direction. With given voltages or currents, respectively, we receive the following equations for $G_\mathrm{p}(t)$:

$$G_\mathrm{p}(t) = \frac{p_\Sigma(t)}{u_\Sigma^2(t)} = \frac{\boldsymbol{u}(t) \cdot \boldsymbol{i}(t)}{\boldsymbol{u}(t) \cdot \boldsymbol{u}(t)} 3.18 \ ; \quad G_\mathrm{p}(t) = \frac{i_\Sigma^2(t)}{p_\Sigma(t)} = \frac{\boldsymbol{i}(t) \cdot \boldsymbol{i}(t)}{\boldsymbol{u}(t) \cdot \boldsymbol{i}(t)} \ . \tag{3.19}$$

In the special case of given voltages the vector of power currents is given by

$$\boldsymbol{i}_\mathrm{p}(t) = G_\mathrm{p}(t) \cdot \boldsymbol{u}(t) \tag{3.20}$$

and the vector of powerless currents by

$$\boldsymbol{i}_\mathrm{ix}(t) = \boldsymbol{i}(t) - i_\mathrm{p}(t) \ . \tag{3.21}$$

As already seen in (3.13), the power associated with the powerless currents is zero, equivalent to the scalar product of the vector of voltages and the vector of powerless currents being zero

$$\boldsymbol{u}(t) \cdot \boldsymbol{i}_\mathrm{ix}(t) = 0 \ , \tag{3.22}$$

demonstrating that the vector of powerless currents is perpendicular to the vector of voltages. With (3.20) it is obvious that the vector of powerless currents is also perpendicular to the vector of power currents. The vectors of terminal currents, power currents and powerless currents, linked by (3.21), form a triangle in the n-dimensional vector space. The angle between the vector of power currents i_p and the vector of powerless currents i_ix is a square one, the vectors are perpendicular. The magnitudes of the vectors are therefore linked by

$$\|\boldsymbol{i}(t)\|^2 = \|i_\mathrm{p}(t)\|^2 + \|\boldsymbol{i}_\mathrm{ix}(t)\|^2 \ . \tag{3.23}$$

3.5 Space vectors (Park vectors) and Hyper space vectors

Currents and voltages used in this lecture are zero-sum quantities. It has already been discussed that (*n-1*) of the n quantities already contain all the information needed. It is therefore possible to reduce the dimension of the vector space introduced in 3.4 from n to (n-1). This reduction is particularly useful in case of three or four terminal circuits. In these cases the reduction of calculation effort is important and it becomes more easy to display currents and voltages in a graphical way. In case of three terminal circuits a plane instead of a three-dimensional space is sufficient, in case of four terminals a three-dimensional space can be used instead of four dimensions. A three-dimensional space can be visualised and understood quite effectively, a four-dimensional space poses big problems.

The transformation used for this reduction has to treat all the components of the vector equally, otherwise the simple equations derived up to now do not remain valid. In case of three-terminal circuits - which are most important for practical applications - the space vector (or Park vector) fulfils these requirements [22]. In case of four terminal circuits the zero-sequence component of the Park transformation (or the space vector transformation) no longer fulfils the requirement of treating all terminals equally. Therefore in this case an extension of this transformation has to be used, for example the hyper space vector transformation [23].

3.6 "Powerless" apparent power

The vector of power currents is associated with the total instantaneous power p_Σ of the circuit (3.8, 3.10, 3.19, 3.20). The vector of terminal currents is associated with the instantaneous apparent power s_Σ (3.5, 3.15). If we multiply (3.23) with the square of the collective instantaneous value of voltages, which is the square of the norm - or magnitude - of the voltage vector,

$$\begin{array}{ccccc} (\|\boldsymbol{i}(t)\|\cdot\|\boldsymbol{u}(t)\|)^2 & = & (\|\boldsymbol{i}_\mathrm{p}(t)\|\cdot\|\boldsymbol{u}(t)\|)^2 & + & (\|\boldsymbol{i}_\mathrm{ix}(t)\|\cdot\|\boldsymbol{u}(t)\|)^2 \\ s_\Sigma^2(t) & = & p_\Sigma^2(t) & + & p_\mathrm{ix}^2(t) \end{array} \tag{3.24}$$

we can formally define the "powerless" apparent power $p_\mathrm{ix}^2(t)$, which links instantaneous apparent power and total instantaneous power. This is a formal definition, because the power associated with the powerless currents and therefore with the powerless apparent power is always zero - the powerless apparent power is a power-like quantity associated with no total power. The naming "powerless apparent power" in some way seems ridiculous - but it clearly describes its meaning - and underlines that only currents, but not apparent powers have a clear physical meaning.

The reason for such a definition is a practical one: The definition allows to weigh the instantaneous nonactive current based on the voltages of the circuit and compare it to the apparent power and the total instantaneous power. It is a measure for the size and costs of a compensator built to compensate powerless currents: It must be connected to a certain voltage level and carry currents of a certain magnitude.

It should explicitly be note that "instantaneous nonactive power", although erroneously used in a previous version of this text, is not a suitable name for this power-like quantity. It is already reserved for "instantaneous value of the nonactive power". Nonactive power is a power quantity defined under periodic conditions in subsection 4.6, which certainly has a time characteristic describing its instantaneous values.

3.7 Sub-Conclusion for section 3

Because of the importance of the concepts presented in this section, the main aspects concerning the equivalent power conductance $G_p(t)$ in connection with (3.5) are presented here in other words and from different points of view:

► A given instantaneous power is reached with minimal instantaneous apparent power then and only then, if the n currents of the system and the n associated voltages, measured against the virtual star point, fulfil the condition given by (3.6) or - using vectors - (3.20).

► If the instantaneous power and the voltages of the system are given, as in most practical situations, then the currents leading to minimal instantaneous apparent power can be derived from (3.6) or - using vectors - (3.20) with G_p resulting from (3.8).

- If the collective instantaneous values of currents and voltages are known - for example resulting from dimensioning the components of the system - the maximum of the instantaneous power which can be reached is identical with the instantaneous apparent power. This maximum is reached only if (3.6) or - using vectors - (3.20) is fulfilled.

- Let us assume that the system is fed from ideal voltage sources by nonideal conductors. Furthermore, the resistances of the nonideal conductors are assumed to be equal with value R. This is fulfilled in most practical situations. The total losses in the conductors in every instant are then given by $\sum_{\upsilon=1}^{n} [R \cdot i_\upsilon^2(t)] = R \cdot \sum_{\upsilon=1}^{n} i_\upsilon^2(t) = R \cdot i_\Sigma^2(t)$. These losses are minimised, if (3.6) or - using vectors - (3.20) is fulfilled with G_p resulting from (3.8), because then the collective instantaneous value of currents $i_\Sigma^2(t)$ is minimal with respect to the given voltages and demanded power.

- For practical applications, the definition of current components is much more important and directly linked to the phenomena under analysis. Power components (3.24) only weigh the current components with the collective instantaneous value of voltages, which may be of interest for dimensioning or billing purposes. For compensation purposes, currents have to be changed if the voltages are given or voltages, if the currents are given in order to reach optimal energy transfer.

4 Currents and powers under periodic conditions

While in section 3 instantaneous quantities without any restrictions concerning their waveforms have been treated, this section is dedicated to periodic quantities which all have the same period T:

$$g(t) = g(t - T) \ . \tag{4.1}$$

It should be noted that periodic is more general than sinusoidal. The waveforms are again not restricted in any way except that they have to be periodic.

4.1 Active power

The active power of a multi-terminal circuit is given by the mean value of its total instantaneous power:

$$P_\Sigma = \frac{1}{T} \int_T p_\Sigma(t)\, dt = \frac{1}{T} \int_T \boldsymbol{u}(t) \cdot \boldsymbol{i}(t)\ dt \ . \tag{4.2}$$

Under the assumption of periodicity the active power is constant. If this definition is applied to nonperiodic quantities, the result depends on the time t and the chosen period T.

4.2 Cauchy-Schwarz inequality defines active conductance and apparent power

The Cauchy-Schwarz inequality (3.3, 3.5) known from section 3 is now extended by integrals used for mean-value calculation based on the period T:

$$\sqrt{\frac{1}{T} \int_T \sum_{\upsilon=1}^{n} i_\upsilon^2(t)\ dt \cdot \frac{1}{T} \int_T \sum_{\upsilon=1}^{n} u_{\upsilon 0}^2(t)\ dt} \geq \left| \frac{1}{T} \int_T \sum_{\upsilon=1}^{n} [i_\upsilon(t) \cdot u_{\upsilon 0}(t)]\ dt \right| \tag{4.3}$$

Using collective rms values (2.6) and active power (4.2) we receive

$$S_\Sigma = \sqrt{i_{\Sigma,\mathrm{eff}}^2 \cdot u_{\Sigma,\mathrm{eff}}^2} = i_{\Sigma,\mathrm{eff}} \cdot u_{\Sigma,\mathrm{eff}} \geq |P_\Sigma|. \tag{4.4}$$

leading directly to the power factor λ given by

$$\lambda = |P_\Sigma| / S_\Sigma \ , \quad 0 \leq \lambda \leq 1 \tag{4.5}$$

which is one single factor characterising the overall quality of energy transfer for a system. Of course the power factor can not tell everything about a system described by an infinite number of parameters, but it gives a good first impression of a system. If the power factor is close to unity, energy transfer is nearly optimal - if it is zero, no energy transfer occurs, all currents are completely superfluous.

If we now look for the condition which ensures equality in (4.4), meaning unity power factor, we find that now

$$i_\upsilon(t) = k \cdot u_{\upsilon 0}(t) = G \cdot u_{\upsilon 0}(t) \quad \forall\ \upsilon \in \{1 \cdots n\} \ \ \forall\ t \in T \tag{4.6}$$

must be valid. Instead of the time variant instantaneous power conductance G_p now a constant active power conductance G has to link the set of currents with the set of voltages. If this definition is used for controlling a compensator, an equivalent load results which again is discussed in subsection 5.3.

In a first step we assume again that the voltages are given. This assumption is acceptable in most practical applications. If energy transfer shall be optimal, the currents then have to fulfil (4.6). Otherwise the existing currents can be changed such, that (4.6) is fulfilled and the total power remains unchanged, leading to a reduced collective value of currents. Again the theory is constructive. The total instantaneous power under the condition of (4.6) is

$$\begin{aligned} P_\Sigma &= \frac{1}{T}\int_T p_\Sigma(t)\,dt = \frac{1}{T}\int_T \sum_{\upsilon=1}^{n} [u_{\upsilon 0}(t)\cdot i_\upsilon(t)]\,dt = \frac{1}{T}\int_T \sum_{\upsilon=1}^{n} [u_{\upsilon 0}(t)\ \cdot G\cdot u_{\upsilon 0}(t)]\,dt \\ &= G\frac{1}{T}\int_T \sum_{\upsilon=1}^{n} u_{\upsilon 0}^2(t)\,dt = G\cdot u_{\Sigma,\mathrm{eff}}^2 \end{aligned} \quad . \qquad (4.7)$$

We directly receive the optimal value of the active conductance G

$$G = \frac{P_\Sigma}{u_{\Sigma,\mathrm{eff}}^2}\ . \qquad (4.8)$$

In a similar way G can be defined if the currents are given:

$$G = \frac{i_{\Sigma,\mathrm{eff}}^2}{P_\Sigma}\ . \qquad (4.9)$$

It should explicitly be noted that the active conductance G in general is **not** the mean value of the equivalent instantaneous power conductance $G_\mathrm{p}(t)$. Although e.g. (3.8) and (4.8) have similar structure and (4.8) results from (3.8) by taking mean values, the mean value of nominator and denominator has to be taken **separately!** If, however, the collective instantaneous value of voltages or currents, respectively, is constant, then G is equal to the mean value of $G_\mathrm{p}(t)$.

4.3 Waveform of active power

This aspect is treated as a special topic, because it is often erroneously assumed that active power, at least with systems fed by three or more conductors, is constant. People tend to believe that every deviation from constant power leads to nonactive power. This is not true in the general case, but only if the collective instantaneous value $u_\Sigma^2(t)$ of voltages (or $i_\Sigma^2(t)$ of currents) is constant. This follows from (3.8) and (3.9) if we replace the instantaneous power conductance $G_\mathrm{p}(t)$ with the active conductance G and solve for the total instantaneous power under these optimal conditions - which include $i_\Sigma(t) = G\cdot u_\Sigma(t)$:

$$p_\Sigma(t) = \frac{i_\Sigma^2(t)}{G} = G\cdot u_\Sigma^2(t)\ . \qquad (4.10)$$

It is obvious that - even with constant factor G meaning optimal energy transfer - the instantaneous value of active power varies with time, if the collective instantaneous values of voltages and currents vary with time.

For circuits with two conductors only and fed by sinusoidal voltage this is very well known: Even with an ohmic resistor connected, power varies with twice the fundamental frequency of the voltage and current. If such a circuit is fed with DC voltage, no power oscillation occurs. In both cases - because of the resistor connected - energy transfer is by definition (and by the considerations presented in this lecture) optimal. This has nothing at all to do with the pure fact that power oscillates. An intrinsic power oscillation is given by the variation of the collective instantaneous values, only deviations from this intrinsic oscillation are linked with nonactive currents. This is discussed further in the subsections 4.5 and 5.

In normal operation of three-phase energy supply systems, however, the collective instantaneous value of voltages is constant. No intrinsic power oscillation is given. In this important case power is constant in case of purely active currents.

4.4 Active currents and nonactive currents

Let us again assume that the voltages are given. Then the optimal set of (instantaneous) currents - called (instantaneous) active currents i_a - follows from using the active conductance G derived in (4.8)

$$i_{a\upsilon}(t) = G \cdot u_{\upsilon 0}(t), \tag{4.11}$$

using vectors

$$\boldsymbol{i}_{a}(t) = G \cdot \boldsymbol{u}(t)\ . \tag{4.12}$$

It should be noted that in general voltages and currents are functions of time. Therefore active currents vary with time (having instantaneous values) - but strictly proportional to the associated voltages. The term "instantaneous active current" should be restricted to the instantaneous value of an active current. In literature, however, it is often misleadingly associated with the power currents defined in section 3.

If a difference exists between the set of active currents $i_{a\upsilon}(t)$ and the set of terminal currents $i_{\upsilon}(t)$, then energy transfer is not optimal. Active currents - with reduced collective rms value - are sufficient to transfer the same energy. Losses in the feeding conductors can be reduced (assuming equal resistances in all feeding conductors).

The difference between the set of active currents $i_{a\upsilon}(t)$ and the set of terminal currents $i_{\upsilon}(t)$, if it exists, is called set of nonactive currents $i_{x\upsilon}(t)$

$$i_{x\upsilon}(t) = i_{\upsilon}(t) - i_{a\upsilon}(t)\ , \tag{4.13}$$

in other words: The terminal currents are the sum of power currents and nonactive currents:

$$i_{\upsilon}(t) = i_{a\upsilon}(t) + i_{x\upsilon}(t)\ , \tag{4.14}$$

using vectors

$$\boldsymbol{i}_{x}(t) = \boldsymbol{i}(t) - \boldsymbol{i}_{a}(t)\ ;\ \ \boldsymbol{i}(t) = \boldsymbol{i}_{a}(t) + \boldsymbol{i}_{x}(t)\ . \tag{4.15}$$

The total active power associated with the nonactive currents is zero, because the active currents were defined such, that they transfer the same total energy during one period as the terminal currents:

$$\frac{1}{T}\int_{T} \boldsymbol{u}(t)\cdot\boldsymbol{i}_{x}(t)\,dt = \frac{1}{T}\int_{T} \boldsymbol{u}(t)\cdot\boldsymbol{i}(t)\,dt - \frac{1}{T}\int_{T} \boldsymbol{u}(t)\cdot\boldsymbol{i}_{a}(t)\,dt = P_{\Sigma} - P_{\Sigma} = 0 \tag{4.16}$$

The active currents are proportional to the voltages with constant factor of proportion (4.6). The integral of nonactive currents times voltages over one period (the active power of the nonactive currents) is zero (4.16). Therefore active currents and nonactive currents are orthogonal sets of functions with respect to the integration interval given by the period T.

4.5 Variation currents

Active currents are always proportional to the set of associated voltages. Therefore they are always power currents. Power currents, however, are not necessarily active currents. They can be subdivide into a set of active currents and a remaining set of currents called variation currents $i_{v\upsilon}(t)$ [24]:

$$i_{p\upsilon}(t) = i_{a\upsilon}(t) + i_{v\upsilon}(t)\ ;\ \ \boldsymbol{i}_{p}(t) = \boldsymbol{i}_{a}(t) + \boldsymbol{i}_{v}(t)\ . \tag{4.17}$$

The nonactive currents defined in subsection 4.4 can also be subdivided into two components. One component is given by the set of powerless currents, the other again by the set of variation currents:

$$i_{x\upsilon}(t) = i_{ix\upsilon}(t) + i_{v\upsilon}(t)\ ;\ \ \boldsymbol{i}_{x}(t) = \boldsymbol{i}_{ix}(t) + \boldsymbol{i}_{v}(t)\ . \tag{4.18}$$

The set of variation currents can therefore be expressed in two different ways: Based on a decomposition given by orthogonal sets of functions according to (4.17) or based on sets of quantities representable by perpendicular vectors (4.18):

$$i_{v\upsilon}(t) = i_{p\upsilon}(t) - i_{a\upsilon}(t) = i_{x\upsilon}(t) - i_{ix\upsilon}(t)\ ;\ \ \boldsymbol{i}_{v}(t) = \boldsymbol{i}_{p}(t) - \boldsymbol{i}_{a}(t) = \boldsymbol{i}_{x}(t) - \boldsymbol{i}_{ix}(t)\ . \tag{4.19}$$

From (4.19) two very important properties characterising variation currents result:

- The first difference in (4.19) shows that the set of variation currents is always proportional to the set of voltages, because $\boldsymbol{i}_{\mathrm{p}}$ and $\boldsymbol{i}_{\mathrm{a}}$ - and therefore their difference - are.
- The second difference in (4.19) shows that the set of variation currents is always a set of currents which does not contribute to the total transfer of energy during one period, because neither $\boldsymbol{i}_{\mathrm{x}}$ nor $\boldsymbol{i}_{\mathrm{ix}}$ contribute.

This leads to the conclusion that variation currents always cause total instantaneous power (because they are proportional to the voltages), but never transfer energy during one period. They describe that part of the power currents, which is nonactive.

4.6 Introducing power quantities

Again it is advisable to start with currents when introducing powers. Combining (4.15) and (4.18) we find that

$$i_{\upsilon}(t) = i_{\mathrm{a}\upsilon}(t) + i_{\mathrm{v}\upsilon}(t) + i_{\mathrm{ix}\upsilon}(t)\ ;\quad \boldsymbol{i}(t) = \boldsymbol{i}_{\mathrm{a}}(t) + \boldsymbol{i}_{\mathrm{v}}(t) + \boldsymbol{i}_{\mathrm{ix}}(t)\ . \tag{4.20}$$

The sets of currents on the right side of the equals sign are orthogonal or perpendicular, meaning that their squared rms values can be added:

$$i_{\mathrm{rms}}^2 = i_{\mathrm{a,rms}}^2 + i_{\mathrm{v,rms}}^2 + i_{\mathrm{ix,rms}}^2\ . \tag{4.21}$$

In the same manner as in connection with (3.24) we can multiply (4.21) with the collective rms value u_{rms}^2 of the voltages and derive

$$\begin{array}{ccccccc} (i_{\mathrm{rms}} \cdot u_{\mathrm{rms}})^2 & = & (i_{\mathrm{a,rms}} \cdot u_{\mathrm{rms}})^2 & + & (i_{\mathrm{v,rms}} \cdot u_{\mathrm{rms}})^2 & + & (i_{\mathrm{ix,rms}} \cdot u_{\mathrm{rms}})^2 \\ S_{\Sigma}^2 & = & P_{\Sigma}^2 & + & Q_{\mathrm{v}}^2 & + & Q_{\mathrm{ix}}^2 \end{array} \tag{4.22}$$

linking apparent power, active power, variation power and powerless apparent power. The relation between the rms values of currents seen in (4.21) (not of the currents or their vector representation) and the powers seen in (4.22) can be visualised geometrically in the following diagram [24] :

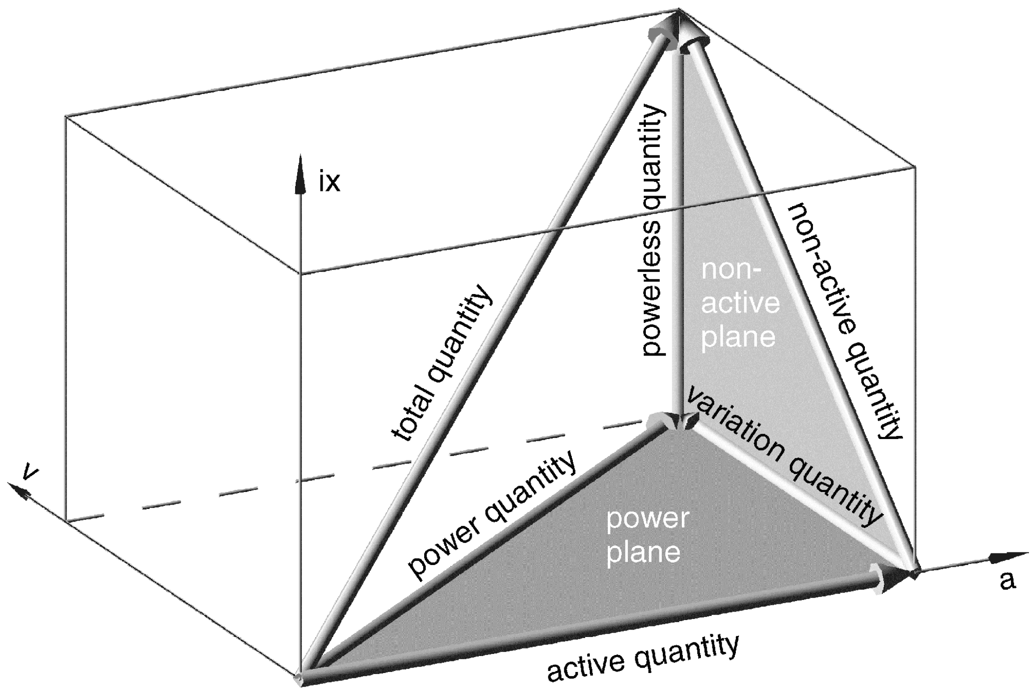


Figure 4.1: Geometrical representation of current and power quantities

It can clearly be seen that active quantity and variation quantity are orthogonal to each other and define the power quantity. Variation quantity and powerless quantity are also orthogonal and lead to the nonactive quantity. The nonactive quantity in turn is orthogonal to the active quantity, both together lead to the total quantity. The total quantity also results from power quantity and powerless quantity.

4.7 Sub-Conclusion for section

Because of the importance of the concepts presented in this section, the main aspects concerning the equivalent power conductance G in connection with (4.6) are presented here in other words and from different points of view:

- A given instantaneous power is reached with minimal instantaneous apparent power then and only then, if the n currents of the system and the n associated voltages, measured against the virtual star point, fulfil the condition given by (4.6) or - using vectors - (4.12).
- If the instantaneous power and the voltages of the system are given, as in most practical situations, then the currents leading to minimal instantaneous apparent power can be derived from (4.6) or - using vectors - (4.12) with G resulting from (4.8).
- If the collective instantaneous values of currents and voltages are known - for example resulting from dimensioning the components of the system - the maximum of the absolute value of the instantaneous power which can be reached is identical with the instantaneous apparent power. This maximum is reached only if (4.6) or - using vectors - (4.12) is fulfilled.
- Let us assume that the system is fed from ideal voltage sources by nonideal conductors. Furthermore, the resistances of the nonideal conductors are assumed to be equal with value R. This is fulfilled in most practical situations. The total losses in the conductors in every instant are then given by $\frac{1}{T}\int_T \sum_{\upsilon=1}^{n} [R \cdot i_\upsilon^2(\tau)]\, d\tau = R \cdot \frac{1}{T}\int_T \sum_{\upsilon=1}^{n} i_\upsilon^2(\tau)\, d\tau = R \cdot i_{\Sigma\mathrm{rms}}^2$. These losses are minimised, if (4.6) or - using vectors - (4.12) is fulfilled with G resulting from (4.8), because then the collective rms value of currents $i_{\Sigma\mathrm{rms}}^2$ is minimal with respect to the given voltages and demanded power.
- For practical applications, the definition of current components is much more important and directly linked to the phenomena under analysis. Power components (4.22) only weigh the current components with the collective rms value of voltages, which may be of interest for dimensioning or billing purposes. For compensation purposes, currents have to be changed if the voltages are given or voltages, if the currents are given in order to reach optimal energy transfer.
- Energy transfer is optimised seen over a period. Therefore all quantities must be known for one period - otherwise G can not be determined.
- Energy transfer is optimised seen over a period. The compensator must therefore provide the difference between the energy of the feeding line and the energy taken by the load. It must be able to store energy. See also subsection 5.4.

5 Compensation issues

We have clearly defined, under which conditions energy transfer to or from an *n*-terminal circuit, here shortly called **load,** is optimal in the sense that active power and apparent power are identical. Assuming given voltages, coming from the power supply network, here shortly called **line,** we also found rules how to determine the currents associated with this optimal energy transfer. If in case of a given load the currents actually flowing in the conductors are not identical with the optimal ones, measures can be taken to optimise the currents, if necessary. Such a procedure is called compensation (of unwanted current components).

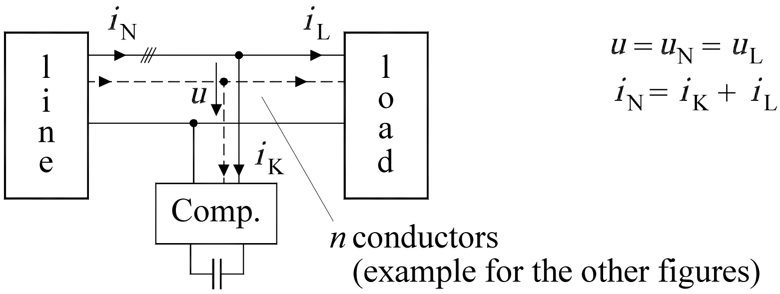
5.1 Compensation using reactive elements (filters)

Several different ways of compensation exist. In most cases uncontrolled "passive" filters constructed mainly of reactive elements (capacitors and inductors), tuned to certain frequencies of interest, damped by resistors with small resistance, are used. Sometimes electromechanical switches are used to switch these elements on and off - depending on the actual load situation. The big advantage of such compensators is that they are comparatively cheap. However, they have a lot of disadvantages:

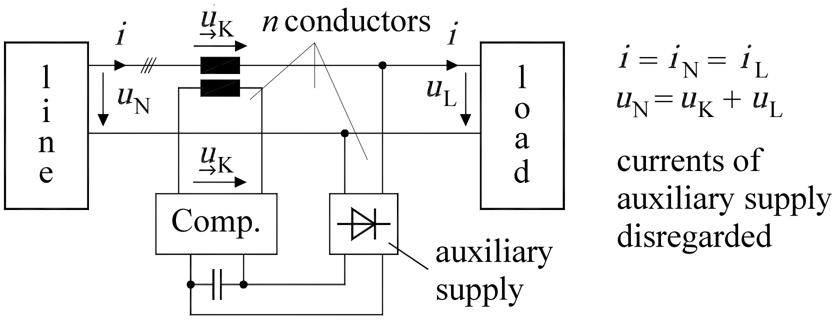
- Due to the slow switches (if there are any at all) they react very slowly to changes in the operation of the circuit whose currents are to be optimised.
- The reactive elements cause not only the desired main resonance with each other, but also resonances together with the feeding power supply system and the load circuit whose currents are to be optimised. These resonances may vary with the state of operation of line and load. These stray resonances are particularly dangerous if they occur in the frequency region of integral multiples of the line frequency or of ripple control signals, because then amplification may occur in a way which may cause the shutdown of energy supply.
- The parameters of the reactive elements change due to ageing effects, influencing the resonances (with possible results as seen above).
- Optimal compensation can only be guaranteed for a limit number of operation points of the circuit under consideration.

5.2 Compensation using controllable power electronic devices

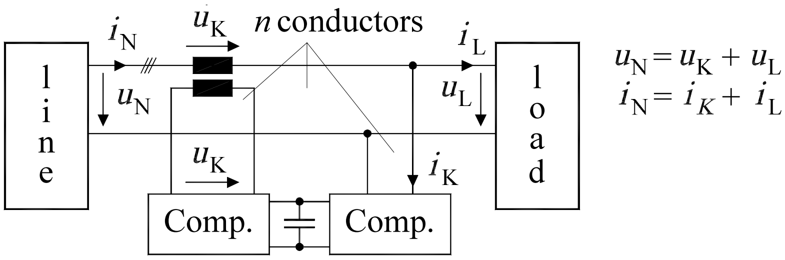
Modern power electronic devices allow for the construction of controllable compensators (often called active filters, although they normally are passive because they do not contain a source of electrical energy). Such controllable compensators are characterised by a relatively high switching frequency provided by power electronic devices. They can be used to overcome all the disadvantages listed above, but they are much more expensive than the passive filters discussed above. The main types of controllable compensators are depicted in the following Fig. 5.1. Usually they are designed for the application in normal three-conductor configurations (with three-terminal systems), but special applications may call for four-wire configurations or two-wire configurations. Therefore the number of conductors is left open in Fig. 5.1, the principle does not depend on the number of conductors.



a: shunt compensator



b: series compensator



c: shunt-series compensator

Figure 5.1: Types of controllable compensators

Fig. 5.1a shows a shunt controllable compensator. This is the most frequently used type. It needs only few elements (the capacitor needed for energy storage is fed by the same converter which is used for the compensation). It can compensate undesirable current components produced by the load. This is most frequently needed, because normally the feeding line impresses voltages and the load reacts causing currents to flow.

Fig. 5.1b demonstrates a series controllable compensator. It is normally used to guarantee sinusoidal load voltages of desired amplitude, if the line voltages are distorted or too low. Using the auxiliary supply - which may also be connected on the line side - low line voltage, especially in case of one-phase faults, can be compensated. It is also possible to take care of sinusoidal line voltages under conditions, where the load impresses non sinusoidal voltages. The auxiliary supply itself can not be used for compensation issues, it consist of a controlled or even uncontrolled line commutated rectifier.

Looking at Fig. 5.1c we find a combination of case a and b. The auxiliary supply is replaced by a shunt controllable compensator, which on the one hand feeds the common capacitor for energy storage and on the other hand can be used to reduce unwanted current component produced by the load. This combination can separate line and load concerning voltage and current distortion completely and handle virtually any situation, but is clearly the most expensive solution.

Leaving the direct link to compensation it is worthwhile to note that the configurations depicted in Fig. 5.1b and Fig. 5.1c can also be used in high-voltage transmission lines to control the flow of electrical energy. For this purpose the series compensator can increase or decrease the line voltage at its location in a controllable way, more quickly and accurately than the usual taps on transformer windings. This increase or decrease of voltage then influences the flow of energy.

5.3 Equivalent load resulting from compensation

If the definitions derived in (3.6) and (4.6) are used to control a compensator such, that the line currents are given by these definitions, then load and compensator together can very graphically be replaced by an equivalent load. This equivalent load is a star connection of n resistors with equal conductance, where each resistor is connected to one of the line conductors with one end and to the common star point with the other end. It is the same configuration which is shown in Fig. 2.1 for the realisation of the virtual star point, but this time the conductance is directly given e.g. by (3.8) or (4.8), respectively, and may be time variant. This visualisation of the compensated load is important, because it clearly shows that all conductors are treated equally and that every conceivable voltage applied is damped, because it is loaded by resistors. Zero sequence voltages and harmonics of any kind are always and equally used to feed the load.

This way of visualisation also shows a drawback of the method: If the load is an active one, feeding energy back to the line, then the conductance of the resistors becomes negative. In this case, all harmonics, zero sequence components etc. contained in the voltages are enhanced and not damped! In this case it is advantageous to feed all the energy back by fundamental positive sequence voltages and - perhaps - load all other voltage components based on the absolute value of the equivalent conductance and so continue to damp undesired voltage components [27, 28].

5.4 Energy storage requirement

In subsection 4.3 the concept of intrinsic power oscillations was already discussed. It means that - depending on the collective instantaneous values of currents - even in case of optimal energy transfer instantaneous power may vary as function of time in a prescribed manner. Variation currents, however, cause deviations from this intrinsic power oscillation. If these variation currents are to be compensated, the energy transferred by them must be stored or provided, depending on the direction of energy flow.

While powerless currents can be compensated on an instantaneous basis without any energy storage capability, this is not possible with variation currents.

The maximal variation of the stored energy of the compensator can be calculated by integrating the variation power $p_{\Sigma\mathrm{v}}(t) = \boldsymbol{u}(t) \cdot \boldsymbol{i}_\mathrm{v}(t)$ associated with the variation currents or - because the powerless currents do not change the total power of the load - by integrating the power associated with the collective instantaneous nonactive currents $p_{\Sigma\mathrm{x}}(t) = \boldsymbol{u}(t) \cdot \boldsymbol{i}_\mathrm{x}(t)$:

$$A_\mathrm{v} = \frac{1}{T} \int_T p_{\Sigma\mathrm{v}}(t)\, dt = \frac{1}{T} \int_T p_{\Sigma\mathrm{n}}(t)\, dt \,; \quad \Delta A_\mathrm{v} = \hat{A}_\mathrm{v} - \overset{\vee}{A}_\mathrm{v} \tag{5.1}$$

where $\hat{A}_\mathrm{v}$ is the peak value and $\overset{\vee}{A}_\mathrm{v}$ the valley value of variation energy A_v.

5.5 Treating dynamics and nonperiodic conditions

In case of dynamics or nonperiodic conditions a suitable period T has to be assumed, which normally is the period of the feeding voltages - or the period the feeding voltages should have, if they are nonperiodic (which is very rarely found in practical situations). The integration process is then realised as sliding mean value calculation, which always takes exactly one assumed period T into consideration and discards everything else. If it is known in advance, that no even-order harmonics are to be considered, then half of the assumed period may be selected for the integration, speeding up the reaction considerably.

Under dynamic conditions it is also possible to select a modified way to calculate the equivalent active conductance, also aimed at speeding up the reaction to dynamics and such reducing the needed energy storage capability of the compensator.

From a theoretical point of view it might seem possible to extend the integration period towards infinity (like moving from Fourier series to Fourier transformation), but from a practical point of view this is not applicable. Quick changes of the load - as momentary events in an infinite time of observation - would lead to practically no change of the equivalent active conductance. The behaviour of the compensated load seen from the feeding line would not change - changes in the flow of energy at the load side would have to be provided completely by the compensator, requiring an enormous amount of stored energy.

6 Examples

6.1 Compensation of powerless and total nonactive currents, dynamics

This example treats the compensation of powerless currents and total nonactive currents. The disadvantages of the powerless current compensation under practical conditions are discussed. We consider an R-L-load connected to the terminals 2 and 3 of the feeding line and choose a shunt compensator to improve the line currents. This configuration can be seen in Fig. 6.1.

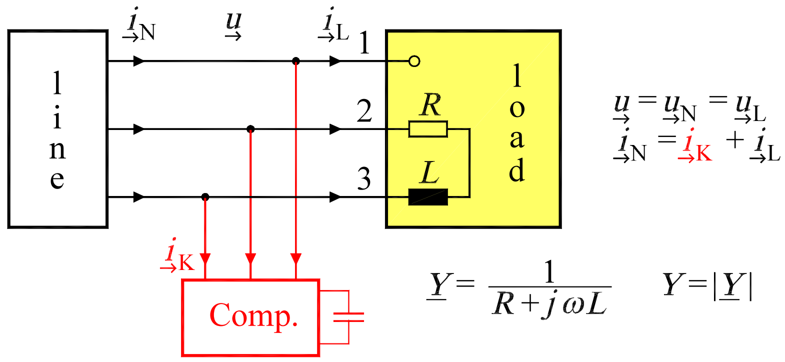


Figure 6.1: Special R-L-load with shunt compensator

We now consider three different modes of operation for the compensator:

- no compensation
- powerless current compensation
- full nonactive current compensation

The results of these compensation methods are depicted in the following figure:

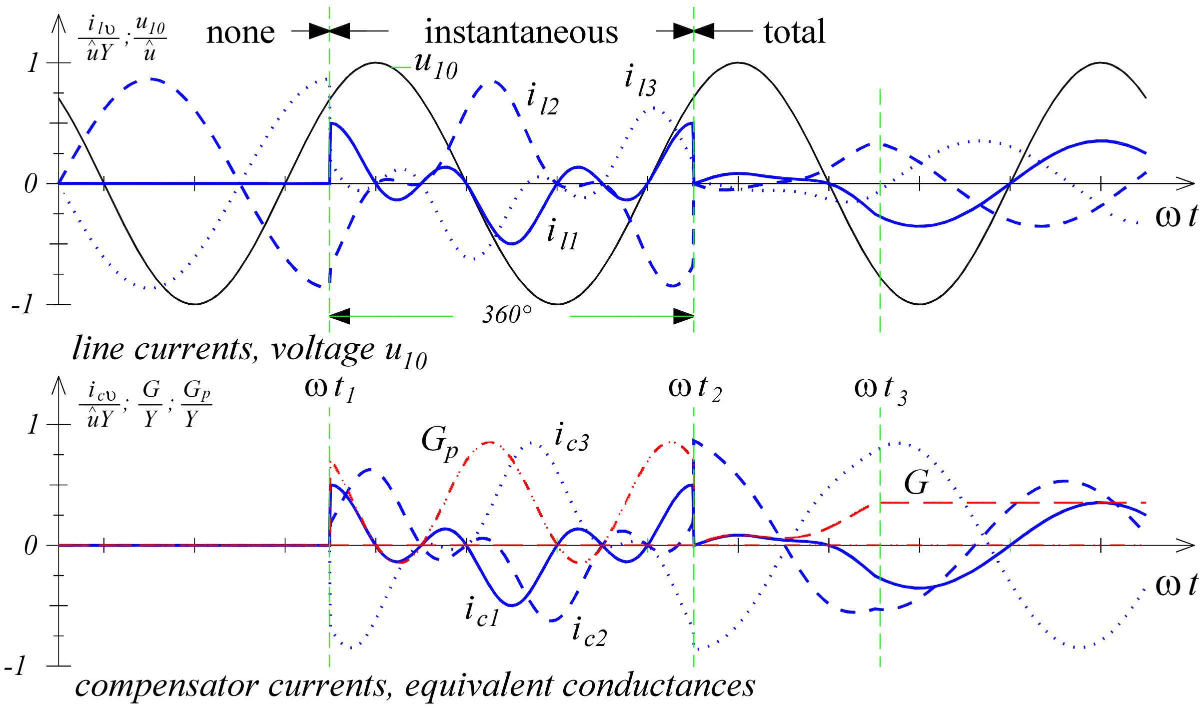


Figure 6.2: Example for effects of compensation methods

6.1.1 No compensation

The upper part of Fig. 6.2 shows the line currents and one of the three line voltages. The voltages are assumed to represent a purely positive-sequence voltage system. The lower part shows the compensator currents and the equivalent conductances which determine the reaction of the compensator. Up to the instant t_1 the compensator is switched off, the upper part of the figure shows the load currents (which are identical to the line currents in this case), the lower part shows that the compensator currents are nil.

6.1.2 Powerless current compensation

At t_1 the compensator is switched on in powerless current compensation mode. The power conductance $G_p(t)$ is evaluated instantly and used to determine the desired line currents. The difference between desired line currents and load currents - the powerless currents - is produced by the compensator as can be seen in the lower part of Fig. 6.2. Due to the variation of the power conductance with time the resulting line currents - upper part of the figure - contain a strong third-order harmonic, which is normally not desired at all [25, 26, 22]. It is normally considered worse than the unsymmetry seen without compensation.

Although powerless compensation is an instantaneous method which does not use the concept of sequence components, in this special case an explanation of the compensation results in case of powerless current compensation can be given based on sequence components and the power oscillation they cause:

The feeding voltages are purely positive sequence. The load is a two-terminal load causing fundamental frequency positive and negative sequence currents. The positive sequence currents are responsible for active power and fundamental frequency reactive power. The fundamental frequency reactive power is associated with a set of zero-sequence currents and compensated at once. This decreases the collective instantaneous value of the line currents.

The fundamental frequency negative sequence currents cause a power oscillation with double fundamental frequency. Such a power oscillation can also be found as the result of a set of positive sequence currents with triple fundamental frequency. The minimal collective instantaneous value of currents is reached, when half of the negative sequence currents is replaced by third-order positive sequence currents causing exactly the same power oscillation - so that the instantaneous power is not changed at all. The remaining set of negative sequence currents and the set of third-order positive sequence currents add geometrically, so that the resulting collective instantaneous value of currents is decreased.

6.1.3 Full nonactive current compensation

At t_2 the compensator is switched on in powerless current compensation mode. The active conductance G is evaluated according to the definitions given before and used to determine the desired line currents. No even order harmonics are to be expected, therefore the integration interval is selected to be only half of the period of the voltages speeding up the process of finding the steady state value of G.

At the beginning of total nonactive current compensation the control calculating G does not know anything about the load and its currents, therefore at the beginning G is equal to zero, all load currents are supplied by the compensator, the line currents are zero. During half a period - the integration interval - the increasing knowledge about the currents taken by the load leads to changes of G, until after half a period at $t_3$the steady state value is reached. From this moment on the currents taken from the line are purely sinusoidal fundamental frequency positive sequence currents.

The difference between desired line currents and load currents - the nonactive currents - is again produced by the compensator as can be seen in the lower part of Fig. 6.2. This time the control of the compensator needs time to detect the correct value of G and energy storage capability to supply and store the difference between the energy supplied by the line and the energy taken by the load.

6.2 Current components in four-terminal system

This example shows the application of the theory introduced above to a four-terminal circuit "load" containing power electronic devices. We consider a load constructed of three sub-loads (Fig. 6.3). Each sub-load is given by a single-phase bridge inverters with capacitive smoothing containing a small smoothing inductance (Fig. 6.4). This type of load represents for example an office building with lots of computers and energy saving lamps inside. The load is fed by three line conductors and a neutral conductor, the voltages feeding the load are given by an ideal positive-sequence three-phase system without any zero-sequence voltage.

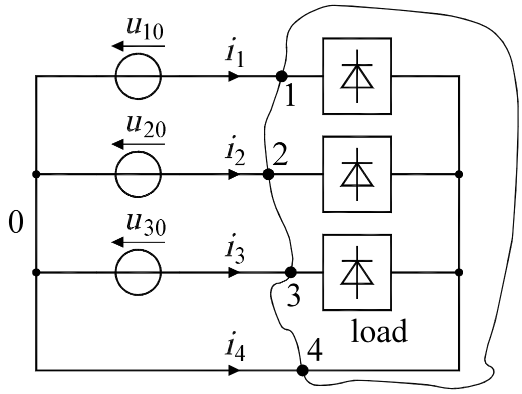


Figure 6.3: Load formed by three identical sub-loads containing nonlinear elements

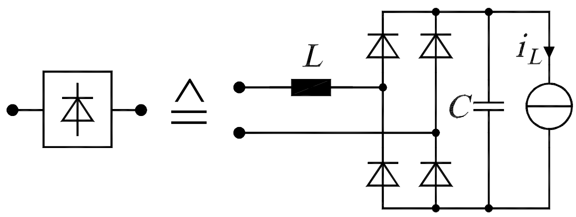


Figure 6.4: Structure of a sub-load (single-phase bridge rectifier with capacitive smoothing)

In Fig. 6.5 the currents of such a type of load are seen. The capacitor is recharged near the positive and negative peak of the respective voltage against the fourth conductor. Because of the phase displacement of the voltages only one of the sub-loads is active at a time. Therefore the current in the neutral conductor (number 4) is the sum of the three sub-load currents. The rms value of this current is given by $\sqrt{3}$ times the rms value of each individual rms value of the currents in the three line conductors (numbers 1 to 3). Normally only the line conductors are protected against overload by fuses. Therefore such a load may overload the neutral conductor without blowing the fuses of the line conductors, which may lead to serious damages.

If we apply the theory developed in this lecture, we immediately note that the neutral conductor current is an instantaneous non active current which may be compensated without energy storage, because there is no zero-sequence voltage component present. The neutral conductor current might even be compensated easily by a simple Z-type transformer, without needing any sophisticated power electronics based compensation device.

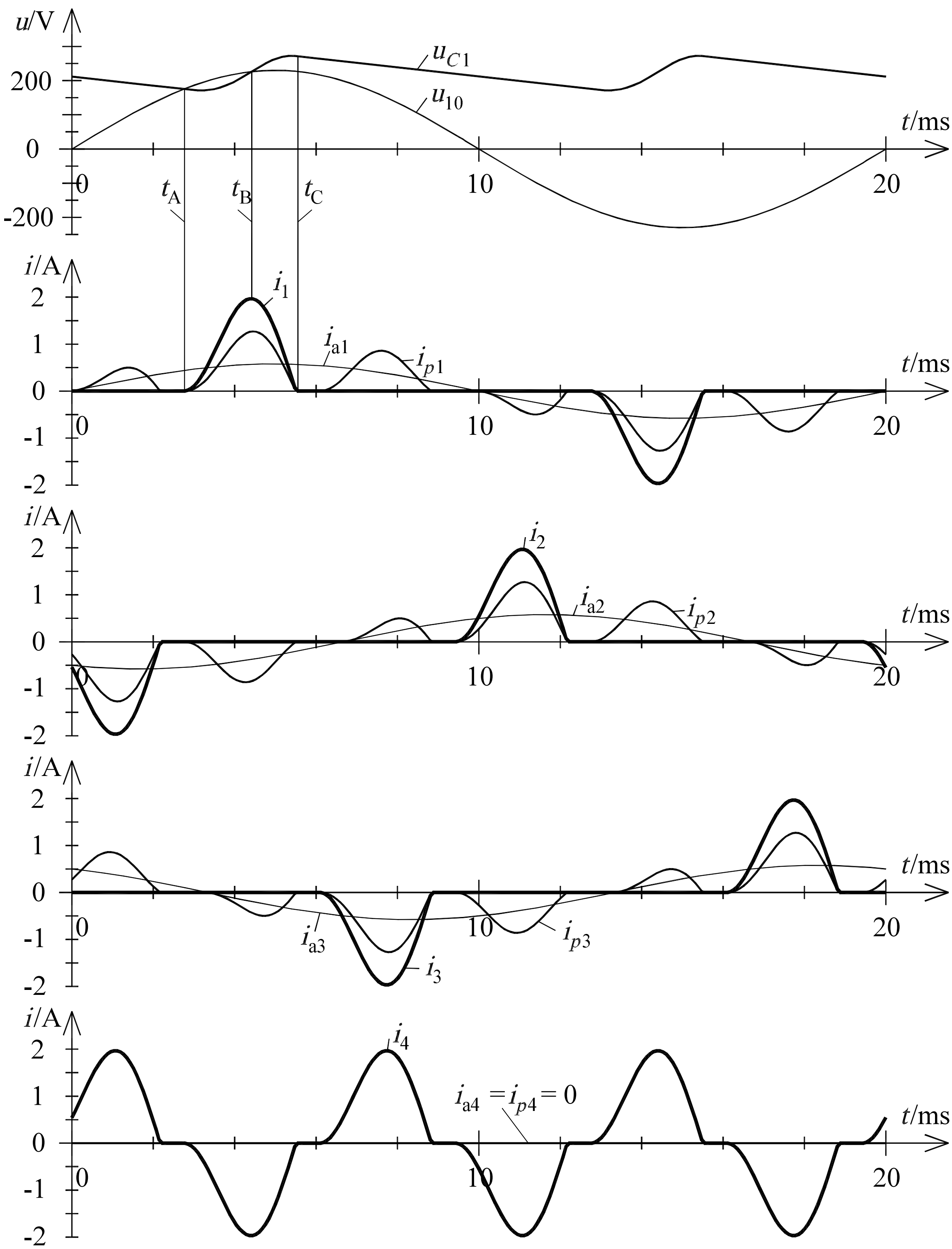


Figure 6.5: Time characteristic of currents and voltages of the nonlinear load

The power currents i_p which would result from compensating powerless currents by a shunt compensator and the active currents i_a resulting from total nonactive current compensation are also shown in Fig. 6.5. This demonstrates that the theory developed in this lecture can easily be used for circuits with an arbitrary number of conductors, either to assess them or to supply set-point values for controlling compensators.

7 Considering harmonics

The theory laid out above leads to power factor unity under all conceivable conditions. This is reached by introducing an equivalent load consisting of n equal constant resistors in star connection. The value of these resistors is given by the equivalent active conductance, which at given voltages results from (4.8). Based on the Cauchy-Schwarz inequalities it is shown that such an equivalent load transfers the energy taken by load during one period with minimal collective quantities of currents and voltages. At given voltages the currents are minimised. Active power and apparent power become identical then and only then, if this special set of currents is used to acquire the energy needed to supply the load. From a theoretical point of view this is the optimal solution, which can for example be realised by a shunt compensator compensating all nonactive currents.

From a practical point of view some question may seem to be open. It is for example commonly assumed that the optimal waveform of currents should be sinusoidal. This seems to be supported by the example presented in connection with Fig. 6.2. There the generation of third order harmonics by instantaneous compensation of the set of powerless currents is explained. This powerless current compensation also minimises the collective quantity of currents, but on an instantaneous basis. The set of currents which fulfils the associated Cauchy-Schwarz inequality and so minimises line losses at equal line resistances generates a undesirable third order harmonic even at sinusoidal voltages.

Might similar effects occur with total nonactive current compensation (at power factor unity) too? The answer is clearly no.

With total nonactive current compensation the optimised line currents are defined by the equivalent load consisting of n equal constant resistors in star connection. The resistors are constant, they are not able to generate any harmonic themselves. This contrasts the resistors of the equivalent load for powerless current compensation, which may be time variant and so cause current harmonics. It is therefore evident that at power factor unity no current harmonics exist which are not already contained in the voltages.

If the voltages feeding the load are nonsinusoidal, the currents at power factor unity will also be nonsinusoidal. However, these nonsinusoidal currents result from constant resistors being fed by nonsinusoidal voltages - they are a reaction to the distorted voltage waveform. It is a well known property of resistors that they always consume electric energy. This property in no way depends on the waveform of the voltage applied - the current waveform is always proportional to the voltage waveform, electric energy flows into the resistor. A Fourier decomposition of voltage and current verifies that every voltage harmonic is loaded by the same resistance independent of the frequency. Therefore energy is transferred to the load not only by the fundamental, but also by all harmonics. The nonsinusoidal currents taken by the load therefore damp the voltage harmonics contained in the feeding voltages and in this way help to improve the waveform of the feeding voltages. They have to be regarded as a positive aspect of the currents - not only because they lead to power factor unity and therefore minimal line losses, but also because they are preferable to sinusoidal waveform in case of nonsinusoidal voltages seen from the effect on line voltage harmonics.

It should be noted that these considerations only include the energetical aspects and conducted EMV, which is reduced. High-frequency voltage harmonics might theoretically lead to high-frequency current harmonics, which in turn might disturb other consumers by radiated electromagnetic fields. However, in practical applications high-frequency current harmonics are due to the limited switching frequency and the limited damping of this switching frequency by filters of commonly used compensation devices.

8 Literature

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